# Simple math for musical ideas

1. Math meets melody

In this short journey we build a childlike song from one of the simplest musical patterns: a linear progression of tones descending step by step until they reach home.. With a few playful math trick, this plain pattern can grow into many different melodies, all related, yet each with its own character.

Think of this text as a primer: a simple doorway into how math and music can play together. Here, we speak only in pitch, letting tones themselves tell the story.

2. The descending linear fifth progression

A linear progression is a voice-leading pattern in tonal music where a line moves stepwise (by step, i.e. by seconds) to span an interval. In this text we focus on a special one: the linear fifth progression - a line that moves step by step to span the distance of a perfect fifth. The important feature is that every tone in between is included, none are skipped.

It can move up (ascending) or down (descending). The descending variant is now of particular importance. Let's give an example of it in C major.
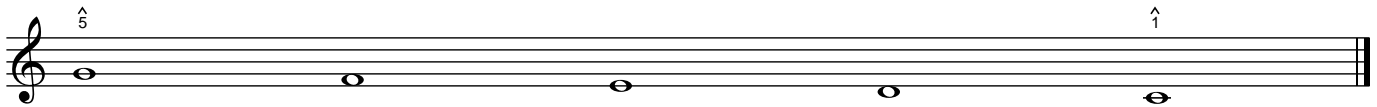
In traditional music theory, the first tone is called the Dominant and the last one the Tonic. The Tonic feels like a point of rest. In other words, a melody can simply end with a descending linear fifth progression, where the last tone feels like a final point.

Composing a song based on this progression is quite easy. But first, I'll outline some basic yet very useful strategies for elaborating on this progression.

3. Shuffling the steps

Our song could end with this melody, which is a linear fifth progression.

To elaborate on the linear fifth progression G-F-E-D-C and create slight variations, we can use permutation. This mathematical tool reorders a sequence of objects - in this case, tones. If we fix G as the first tone and C as the last, we can permute F, E, and D. The possible permutations of F-E-D are: F-D-E, E-F-D, E-D-F, D-F-E, and D-E-F. Two examples:

4a. Inviting a new friend

We start with a simple set of tones: G–F–E–D–C. To make it more interesting, we can add a new tone that wasn't there before. In the key of C major, the note A is a very strong choice. By bringing in A, we don't lose the original tones, but we open up more possibilities for how the melody can move. In math, this way of combining old and new into one group is called a set union.

4b. Saying goodbye to a friend

We start with a set of tones: A–G–F–E–D–C. This time, instead of adding something new, we can make a change by taking one or more tones away. The melody becomes simpler, but it also takes on a new character. In math, this way of removing items from a group is called a set difference.

5. Playing next door (neighbourtones)

Repetition of a tone can bring liveliness to a melody. However, replacing that repetition with an alternative melodic figure can add variety. A neighbourtone is a tone that departs from a stable pitch, moves to another pitch (commonly by step but possibly by a larger interval), and then returns to the original pitch, creating a symmetrical motion away and back. Examples of such neighbourtone alternatives include:

The previous melody can be modified as follows.

6. Forward and back again: a little song

Form in music is often based on repetition. So we can make a 4 bar song simply by repeating the 2 bar linear fifth progression as a prefix.

Let's evaluate this song.

Pros:

- Very clear, easy to follow.

- Creates a childlike, chant-like quality.

- Strong closure, since it twice reaches the tonic (C).

Cons:

- Could be perceived as monotonous

Two identical progressions can sound flat and predictable. Using varied repetition can make them more engaging.

Less monotonous is when we reverse the order of the tones of the prefix section. If we take the descending linear fifth progression: G F E D C and simply reverse the order of pitches, we get C D E F G which is the ascending linear fifth progression.

We could also say: the ascending version is the retrograde of the descending one. This idea of a linear progression and its retrograde is a great strategy to compose a melody, which is more interesting.

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

The idea that a linear progression and its retrograde together form a complete melodic gesture is a thoughtful and effective compositional strategy:

- it creates natural symmetry,

- a clear tension-and-release arc,

- and a melody that is immediately easy to remember, almost like a childlike song.

This is why such motions appear so often in children's songs, folk tunes, and simple melodies: the brain instantly recognizes the stepwise motion and its reversal.

Now we can easily apply the previous mentioned techniques on this symmetrical melody. A few examples.

Basic version - the simple ascending and descending fifth progression, unaltered.

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

Permutation (bar 3) - tones reordered slightly, keeping the same frame.

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

Added friend - the extra tone (A) enriches the line.

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

Goodbye to friends (bar 1), creating a leaner melody.

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

Try combining techniques. What do you think of this example?

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

7. Reversing melody: parts vs. tones

To determine the retrograde of a melody, we can do two things: either reverse the melodic parts, or reverse the order of all the tones.

1. Reversing the melodic parts means first dividing the melody into smaller segments (for example, phrases or motifs), and then reversing the order of those segments while keeping the tones inside each segment unchanged.

2. Reversing the order of all the tones means taking the melody tone by tone and writing them backwards, so the last tone becomes the first, the second-to-last becomes the second, and so on.

Let's apply these techniques on the next melody:

1. Retrograde by reversing parts

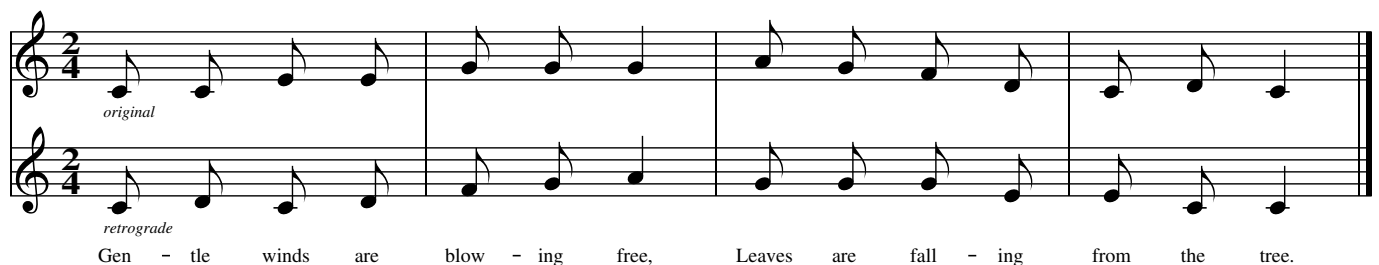Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.
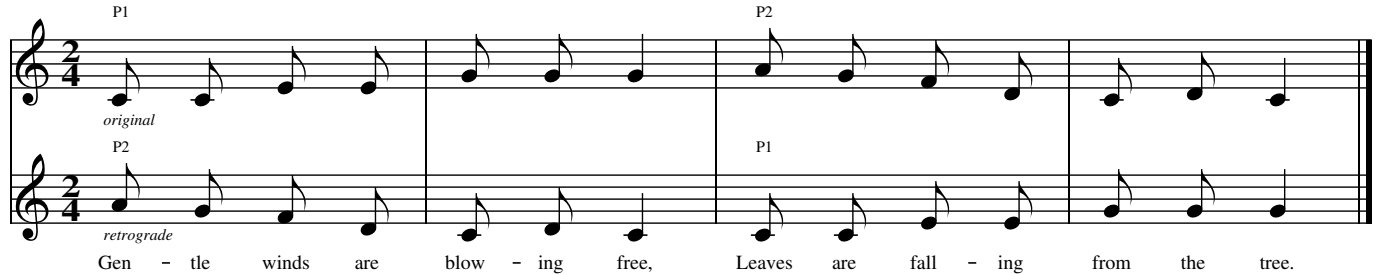
You could say that swapping is at play here: the two segments, P1 and P2, have exchanged their positions within the melody. With some modifications to suit my musical needs, i.e. removing the stop in bar 2:

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

2. Retrograde by reversing all tones

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

With a few changes to fit my musical needs, removing the slow start in bar 1 and softening the prominent A in bar 2:

8. Turning the melody upside down

When we invert a melody, we turn it upside down - as if placing it in front of a mirror. Every interval flips direction: an upward step becomes the same-sized step downward, and a downward step becomes an upward one. For example, if the melody moves up a major third, the inverted version moves down a major third.

However, strict mirroring can sometimes conflict with tonal constraints. In such cases, it may be better to adjust the intervals slightly - for instance, mirroring an upward major third with a downward minor third - to preserve the sense of key.

The result is a melody that reflects the original, as if seen in a mirror. Let's apply this technique on the last melody.

And after adapting this mirror melody to my musical purpose:

It's easy to transpose the mirror melody to a suitable pitch.

9. Musical carousel

In the musical carousel, the notes keep riding in the same circle, but each one gets a turn at the front. To 'rotate' a melody, we simply shift the starting point while keeping the order of notes unchanged. For instance, the line G-F-E-D-C can rotate to F-E-D-C-G, or to E-D-C-G-F, and so on. Every rotation preserves the interval pattern of the original, but each offers a fresh entrance. The result is a variation that stays closely connected to its source, like a familiar tune viewed from a new doorway.

original

rotation first part

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

You may adapt the melody freely. The next example is my suggestion, in which the tones marked with a cross indicate the changes:

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

Of course, the techniques mentioned earlier can also be applied to this melody. For example, I use retrograde - first on the two main parts (with a small modification at the end of the new second part), and then apply both retrograde and mirror to the tones of the new first part (you can check this yourself). The result is:

Gen – tle winds are blow – ing free, Leaves are fall – ing from the tree.

10. Closing words to this primer

From a single stepwise pattern, countless melodies can unfold. By using simple mathematical ideas - adding, removing, shuffling, reversing, mirroring, rotating, and even combining these techniques - the plainest line takes on playfulness and vitality. This is the beauty of stepwise progressions: with just a few small steps, an endless variety of musical ideas comes to life.

This text is a primer: not a full exploration, but a starting point. Its aim is to show how childlike simplicity can open the door to rich musical thinking.